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Harvesting effect a Ratio-Dependent Predator-Prey Model

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Abstract. The predator-prey model is mathematically ratio-dependent, always showing dynamics rich in singularity findings. Intervention in the model with the predator-prey model of exploitation is carried out by taking into account harvesting. The ratio-dependent form of harvesting predator-prey model is a very important research project in ecological and mathematical theory. In this study, we discuss the dynamics of the ratio-dependent predator-prey model, as well as the forms of exploitation in the harvesting variable. The form of harvesting offered is sustainable harvesting or referred to as bioeconomic theory. The consideration is that the bioeconomic system is able to obtain maximum benefits and at the same time has ecosystem sustainability for a very long period of time. The results of the analysis were carried out without harvesting and with harvesting. Each form of treatment produces two stable local equilibrium points. Harvesting is done only at the equilibrium point that results in maximum population growth, to prevent extinction. In the trajectory analysis, the movement of population growth before and after harvesting is also shown. In the prey population, the movement of population growth is decreasing but towards great stability. Meanwhile, the predator population has decreased significantly, after harvesting. This condition is very possible to occur in an ecosystem that depends on the ratio-dependent.

Keyword: Dynamics, Predator-Prey, Ratio-Dependent

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1 Introduction

The predator-prey model in the field of scientific knowledge has advanced tremendously in the last few decades. The development of types of response functions, types of interactions, stage structure, and harvesting model interaction interventions color the novelty of recent research [1]. In the form of mathematical modeling of predator-prey interactions with harvesting interventions, the most studied in applied mathematics knowledge [2]. This is the most dominant theme in the world of mathematics and ecology, because of the universal nature of science. Although the predator-prey theory has shown many advances in research, there are many mathematical and ecological problems that need continuous attention from researchers [3]. The problem of predation interaction and the problem of exploitation efforts at population density seems simple, but it is often challenging and more complex when viewed from the point of view of the reality of life in predator-prey ecosystems [4]. Many research projects involve predation interactions that are consistent with the characteristics of the predation process [5][6][7]. Many of the predator-prey function responses depend on the prey or vice versa, depending on the predator. There are also many studies involving the interaction of the two populations, namely predator and prey species. Interactions like this usually need each other or are interdependent on one another. The following are some of the response functions that depend on predator, prey and predator-prey interdependence interactions.

1.1 $f(x,y)=f(x)$ the function response dependens only on x

1.1.1 Holling Type I

$$f(x)=mx,$$

1.1.2 Holling Type II

$$f(x)=\frac{mx}{x+a},$$

1.1.3 Holling Type III

$$f(x)=\frac{mx^2}{x^2+a},$$

1.1.4 Holling Type IV

$$f(x)=\frac{mx}{x^2+bx+a}. \text{ [8]}$$

1.2 $f(x,y)$ the response of the that it dependens on x and y

1.2.1 Ration-Dependent

$$f(x,y)=\frac{mx}{x+ay},$$

1.2.2 Beddington-DeAngelis

$$f(x,y)=\frac{mx}{ax+by+c}, \text{ [9]}$$

1.2.3 Hassel-Varley

$$f(x,y)=\frac{mx}{ax+by^{\gamma}+c},\ \gamma=\frac{1}{2},\frac{1}{3}.$$

In this research, the novelty focus is given to the interdependent response function, namely the predator and prey species, namely $f(x,y)$, The ration-dependent response function is given to the predator-prey model built with realistic assumptions. Our consideration using the ratio-dependent is to reveal the types of species that have these predation characteristics.

In terms of trajectories, some significant differences between the forms of the response function are as follows,

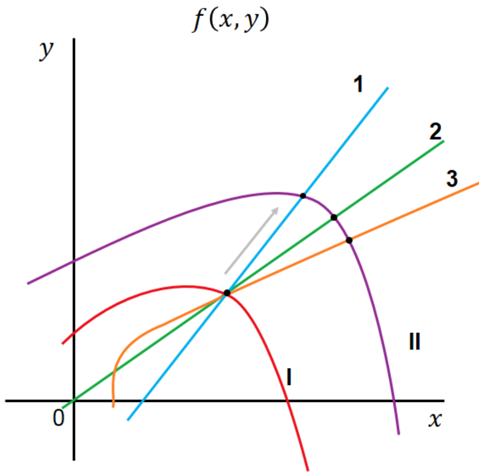

Fig. 1. Trajectories respon function Beddington-DeAngelis, ratio-dependent, and gradual interference trophic functions.

Some species that have the characteristics of a ratio-dependent response function are Cladocerans, Daphnia Magna, and Simocephalus the Elder. The characteristics shown in predation are inter-species dependence [8]. The nature of dependence on the population ratio is the characteristic of this response function. Significant differences in the density of predator populations or prey populations in fact greatly affect the overall sustainability of life in the ecosystem. Such reciprocal relationships between species also occur in the Hassel-Varley (HV), Hassel-Varley-Holling (HVH), and Beddington-DeAngelis forms of response functions [10]. The response function theory is not actually in accordance with the conditions in the field, especially in ecosystem life. Long and controversial debates between the views of ecologists and mathematicians still occur today. But much modeling knowledge has contributed to the development of ecological or biological mathematical sciences.

The ratio-dependence type response function is more suitable to describe the predator-prey interactions that occur. In some situations, when predators must find food in conditions of food sharing or fierce competition for food [11]. Everything becomes a dependency involving mathematical ratios. The growth rate of predators per capita is a function of the ratio of prey to the abundance of predators. The study has been supported by a lot of relevant research in field observations and ecological laboratories. In interactions that are interdependent on the population of each species, the function ratio-dependence response is more appropriate to consider [12]. The involvement of the ratio-dependent in the mathematical model of the population equation will provide a clear picture according to the actual ecosystem conditions. The closer to the actual conditions, the better the mathematical model that is formed.

The predator-prey model of the ratio-dependence form has been developed and richly analyzed for its dynamics. The novelty of the proposed model is as follows,

$$\frac{dx}{dt} = rx\left(1-\frac{x}{k}\right) - \frac{\alpha xz}{x+bz}, \quad \frac{dz}{dt} = \frac{\beta xz}{x+bz} - \delta z, \qquad (1)$$

with the initial conditions of the model (1) $\frac{dx}{dt}=0$ and $\frac{dy}{dt}=0$. Where $x(t)$ and $y(t)$ is the population density of predator and prey, at time t population growth. Parameters is the growth of prey that adopts the intrinsic growth of the species [13]. Parameter k is carrying capacity which is the living area of the ecosystem. The parameter is the catch coefficient of the predation process. The parameter is the conversion rate coefficient of the predator-prey predation process. The parameter is the coefficient of half capturing saturation constant.

Exploitation effort will be given for model (1) with the concept of sustainable exploitation in the model ecosystem. The dynamic analysis will be given to the dynamics model without exploitation, exploitation of the prey population itself, and the predator population itself [14]. In the exploitation of the two populations it is not recommended, because there is a great fear of species extinction in the form of model (1). Model (1) which is intervened by exploitation efforts on prey is as follows,

$$\frac{dx}{dt} = rx\left(1-\frac{x}{k}\right) - \frac{\alpha xz}{x+bz} - H_1\tau x, \quad \frac{dz}{dt} = \frac{\beta xz}{x+bz} - \delta z, \qquad (2)$$

Model (1) which is intervened by exploitation efforts on predators is as follows,

$$\frac{dx}{dt} = rx\left(1-\frac{x}{k}\right) - \frac{\alpha xz}{x+bz}, \quad \frac{dz}{dt} = \frac{\beta xz}{x+bz} - \delta z - H_2\tau z, \qquad (3)$$

The model equations (2) and (3) are accompanied by exploitation efforts, with the condition that,

$H_i\tau = 0$, if no harvesting is done.

$H_i\tau =$ non-negative, if harvesting is done.

Population dynamics studied in the ratio-dependent form have revealed many predator dynamics such as deterministic extinction, the existence of stable boundary cycles, the presence of many attractors, and the presence of stable boundary cycles. The shape of the dynamics of population growth in the geometric trajectories often forms parabolic orbits, elliptic orbits, hyperbolic orbits, or a combination thereof. Changes in the geometry of the trajectories are heavily influenced by the different parameters taken for the mathematical model simulation. The predator-prey population model (1) has been extensively developed in the form of cannibalism, stage structure and other interventions. The novelty focus of model (1) is to involve exploitation interventions in model (1).

2 Method

This research is a type of literature study research, involving relevant research results to build results and discussion. The model developed is a predator-prey model with a selective function ratio-dependence interaction and harvesting response. The basic model developed is model (1), with exploitation intervention in models (2) and (3) [8]. Analysis will be given on all the possibilities that occur in each model [15]. Dimensional analysis is also given to model (1) with all available parameter types. Where all parameters in model (5) are considered with dimensional parameters, presented in the following table,

Table 1. Parameters, Biological Meanings and Unit

Par	Meaning	Unit
x	Prey one population (time-dependent),	$[N]$
z	Predator population (time-dependent),	$[N]$
r	Prey's one intrinsic growth rate,	$[T]^{-1}$
k	Prey's environmental carrying capacity,	$[N]$
α	Capturing rate,	$[T]^{-1}$
β	Conversion rate,	$[T]^{-1}$
b	Half capturing saturation constant,	-
δ	Predators death rate,	$[T]^{-1}$
τ	Catch coefficient,	-
H_1	Prey harvesting,	-
H_2	Predator harvesting.	-

3 Resut and Disscusion

3.1 Equilibrium Analysis

Equilibrium analysis is performed on model (1), by looking at the overall shape of a non-negative equilibrium which is realistic to consider. The differential equation linearization method is carried out for model (1). The overall point of this equilibrium will also determine the exploitation of each species. The differential equation model associated with model (1) is as follows;

$$rx\left(1-\frac{x}{k}\right)-\frac{\alpha xz}{x+bz}=0,$$
$$\frac{\beta xz}{x+bz}-\delta z=0, \quad (4)$$

From the form of model (4) we obtain two forms of non-negative equilibrium points, ie $E_0(x_0,0)$ and $E_1(x_1,z_1)$. Each form E_0 and E_1 is as follows;

$$x_0=k,$$
$$x_1=-\frac{k(-b\beta r+\alpha\beta-\alpha\delta)}{b\beta r},$$
$$z_1=-\frac{k(-b\beta r+\alpha\beta-\alpha\delta)(b-\delta)}{b^2\beta r\delta}.$$

All equilibrium points support the sustainability of the model in an ecosystem. The shape of the equilibrium point E_0 become a point that deserves to be reckoned with the sustainability of prey species. Conditions are different for predatory species where it is not possible to have good population growth. Actually this condition can also take place in ecosystems. It is very possible to occur in an ecosystem without any predator species. one of the goals of this research is to show the sustainability of both species, so that the most realistic thing to consider is the equilibrium point E_1. Equilibrium point E_1 is an equilibrium that allows for further analysis, because of the ever-growing existence of species. Moreover, this research will also take action on the exploitation of each species. Harvesting can only be done in species that have non-negative and non-zero growth. The necessary condition used for harvesting intervention is a stable equilibrium point. point stability E_1 will be analyzed using the Jacobian matrix andRouth-Hurwitz criteria testing, in order to find the corresponding eigenvalues.

The Jacobian matrix corresponding to the equilibrium point is as follows,

$$J_{cob}(E_1)=\begin{bmatrix} j_{11} & j_{12} \\ j_{21} & j_{22}\end{bmatrix},$$

where,

$$j_{11}=r\left(1-\frac{x}{k}\right)-\frac{rx}{k}-\frac{\alpha z}{bz+x}+\frac{\alpha xz}{(bz+x)^2},$$
$$j_{12}=-\frac{\alpha x}{bz+x}+\frac{\alpha bxz}{(bz+x)^2},$$
$$j_{21}=\frac{\beta z}{bz+x}-\frac{\beta xz}{(bz+x)^2},$$
$$j_{22}=\frac{\beta x}{bz+x}-\frac{b\beta xz}{(bz+x)^2}-\delta.$$

From the Jacobian matrix form, the characteristic equation associated with model (4) will be obtained as follows;

$$\lambda^2+b\lambda+c=0, \quad (5)$$

where,

$$b=\frac{b\beta^2\delta+b\beta^2r+b\beta\delta^2+\alpha\beta^2-\alpha\delta^2}{b\beta^2},$$
$$c=\frac{b\beta^2\delta r-\alpha\beta^2\delta-b\beta\delta^2r+2\alpha\beta\delta^2-\alpha\delta^3}{\beta^2 b},$$

It is clear that two eigenvalues will be obtained which become the roots of the characteristic equation (5). The eigenvalues corresponding to the characteristic equation (5) are,

$$\lambda_1=\frac{1}{2b\beta^2}\left(N+\sqrt{M}\right),$$
$$\lambda_2=\frac{1}{2b\beta^2}\left(N-\sqrt{M}\right),$$

where,

N and M, is a non-negative constant corresponding to each of the eigenvalues. on form λ_1 and λ_2 it is clear that each eigenvalue fulfills the form of the Routh-Hurwitz criteria, $\lambda_1<0$ and $\lambda_2<0$. Thus the equilibrium point E_1 can be expressed as a stable asymptotic local equilibrium point. Ecologically, it can also be said that populations of predators and prey that live in an ecosystem are able to coexist for a long time. After this is obtained, then exploitation efforts can be carried out, because harvesting interventions indirectly reduce the number of each population. Harvesting business analysis will be carried out at the numerical simulation stage, in order to facilitate the calculation of the harvesting scheme for each species.

3.2 Equilibrium analysis with prey harvesting

Equilibrium analysis is carried out using model (2) which involves intervention of exploitation efforts on prey. The differential equations that are linearized are as follows;

$$rx\left(1-\frac{x}{k}\right)-\frac{\alpha xz}{x+bz}-H_1\tau x=0,$$
$$\frac{\beta xz}{x+bz}-\delta z=0, \quad (6)$$

From the shape of the model (6), a realistic non-negative equilibrium shape for harvesting is obtained, namely equilibrium $E_1(x_1,z_1)$. Form E_1 associated in model (6) is as follows;

$$x_1=-\frac{k(b\beta\tau H_1-b\beta r+\alpha\beta-\alpha\delta)}{b\beta r},$$
$$z_1=-\frac{k(b\beta\tau H_1-b\beta r+\alpha\beta-\alpha\delta)(b-\delta)}{b^2\beta r\delta}.$$

Point $E_1(x_1,z_1)$ associated with model (6) will be analyzed with the harvesting variable. The optimum profit obtained will also be analyzed in model (6).

3.3 Equilibrium Analysis with Predator Harvesting

Equilibrium analysis is carried out using model (3) which involves the intervention of exploitation efforts on predators. The differential equations that are linearized are as follows;

$$rx\left(1-\frac{x}{k}\right)-\frac{\alpha xz}{x+bz}=0, \quad (7)$$

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$\frac{\beta xz}{x+bz}-\delta z-H_2\tau z=0$,

From the shape of the model (7), a realistic non-negative equilibrium shape for harvesting is obtained, namely equilibrium $E_1(x_1,z_1)$. Form E_1 associated in model (7) is as follows;

$$x_1=-\frac{k(-\alpha\tau H_2-b\beta r+\alpha\beta-\alpha\delta)}{b\beta r},$$

$$z_1=-\frac{k(-\alpha\tau H_2-b\beta r+\alpha\beta-\alpha\delta)(-\tau H_2+\beta-\delta)}{b^2\beta r(\tau H_2+\delta)}.$$

Equilibrium point $E_1(x_1,z_1)$ which is associated with model (7) will be analyzed for harvesting variables. Harvesting business is analyzed to obtain the optimum profit obtained.

3.4 Numerical Simulation

Model (1), model (2) and model (3) will be given a simulation to see the basic scheme of population growth for each species. Numarical simulation is also to see the maximum profit obtained from the harvesting business carried out. Parameters in each form of the model are given from several valid and relevant references. Other parameters are also taken from relevant assumptions according to real ecosystem conditions. Parameters are given mathematically as follow, $r=1.32$, $k=100$, $\alpha=0.05$, $b=0.8$, $\delta=0.03$, $\beta=0.05$, and $\tau=0.5$.

Model (1), which is simulated with our harvesting, is given parameters to be,

$$1.32x\left(1-\frac{x}{100}\right)-\frac{0.05xz}{x+0.5z}=0,$$
$$\frac{0.05xz}{x+0.5z}-0.03z=0. \quad (8)$$

From form (8), the equilibrium point is obtained $E_1(96.96969697,129.2929293)$. Equilibrium point E_1 will be analyzed with the Jacobian matrix and Routh-Hurwitz criteria.

Equilibrium point E_1 It is realistic to consider continuous stability testing. A positive equilibrium value is the main basis for intervention in exploitation business behavior. From the two corresponding equilibrium points it is clear that the growth of predator species is more significant than the growth of prey species. It should be noted that the equilibrium point E_1 obtained from model (8) is a model without harvesting.

The characteristic equation that emerges from the equilibrium point is,

$$\lambda^2+1.268000000\lambda+0.01536000000=0, \quad (9)$$

The eigenvalues corresponding to the characteristic equation (9) are respectively $\lambda_1=$-1.25576844564516 and $\lambda_2=$-0.0122315543548386.

It is clear that for model form (1) it has an asymptotic local stable equilibrium point, ie $E_1(96.96969697,129.2929293)$.

case 1: harvesting with population prey

Model (2) was analyzed with harvesting for prey species. Exploitation business intervention is given to prey species that will be exploited to obtain an estimated profit. after being given the parameters, the shape of model (2) becomes as follows,

$$1.32x\left(1-\frac{x}{100}\right)-\frac{0.05xz}{x+0.5z}-0.4H_1x=0,$$
$$\frac{0.05xz}{x+0.5z}-0.03z=0.. \quad (10)$$

From form (10), the shape of the equilibrium point is obtained, namely

E_1(-30.30303030H_1+96.96969697,-40.40404040H_1+129.2929293)

. The equilibrium point that appears each has a dependence on the harvesting prey variable. The next step is to involve the harvesting function that the author considers in sustainable harvesting. The profit function of the equilibrium point E_1(-30.30303030H_1+96.96969697), (-40.40404040H_1+129.2929293) is,

$$\psi(H_1)=100H_1(-30.3030303H_1+96.96969697)$$
$$-50H_1 \quad (11)$$

From the form of the profit function (11) in harvesting for prey, a derivative solution is obtained $\frac{\partial}{\partial H_1}(\psi(H_1))$ on H_1 which has size $H_1=1.59175$. The harvesting effort carried out is $H_1=1.59175$, get a profit of $\psi(H_1)=7677.782008$. So in the population prey exploitation business, a profit of $\psi(H_1)=7677.782008$.

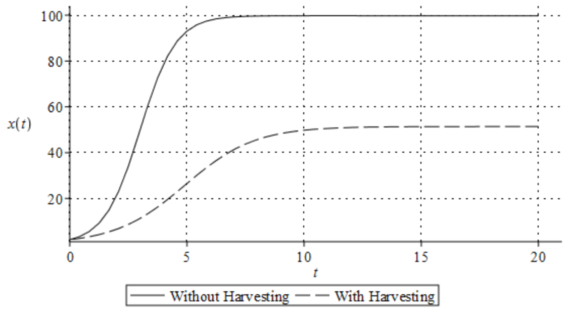

Fig. 2. Trajectories Prey Species.

case 2: harvesting with population predator

Model (2) was analyzed with harvesting for predator species. Exploitation business intervention is given to predatory species to be exploited to obtain an estimated profit. After the parameters are given, the shape of model (2) becomes as follows,

$$1.32x\left(1-\frac{x}{100}\right)-\frac{0.05xz}{x+0.5z}=0,$$
$$\frac{0.05xz}{x+0.5z}-0.03z-0.4H_2z=0. \quad (12)$$

From form (12), the shape of the equilibrium point is obtained, namely $E_2(60.60606061H_2+96.96969697,)$.

$\frac{48.48484848(5H_2+8)(20H_2-1)}{40H_2+3}$.The equilibrium point that appears each has a dependence on the harvesting prey variable. The next step is to involve the harvesting function that the author considers in sustainable harvesting. The profit function of the equilibrium point E_2 is,

$$\psi(H_2)=\frac{48.48484848(5H_2+8)(20H_2-1)}{40H_2+3}-50H_2. \quad (13)$$

From the form of the profit function (13) in harvesting for prey, a derivative solution is obtained $\frac{\partial}{\partial H_2}(\psi(H_2))$ on H_2 which has size $H_2=0.02189109027$. The harvesting effort carried out is $H_2=0.02189109027$, get a profit of $\psi(H_2)=123.7573838$. So in the exploitation of the population predator, a profit of $\psi(H_2)=123.7573838$.

In terms of the trajectories of the population effort, you can see the following trajectories:

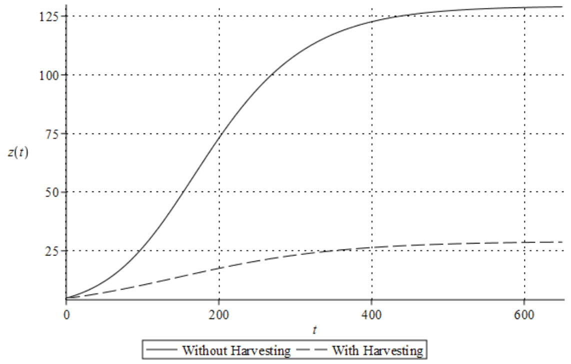

Fig. 3. Trajectories Predator Species.

The two exploitation efforts on prey and predator species each have their own characteristics. However, when compared to the two, the most profitable for exploiters is the prey species. Conditions like this can occur in all species of living things [16]. Even though quantitatively the equilibrium conditions are stable, the number of prey populations is less than the number of predator species [17]. So in a given model with less population harvesting, it turns out to provide more benefits. On the other hand, in the case of a population predator, with a large number of populations, it turns out that the profit generated from the exploitation effort is small. After all harvesting efforts have been carried out, the number of stable population growth tends to decrease. This decrease ecologically and economically provides a good picture. Because the population is rare after harvesting, the number of population increases, except for special conditions.

4 Conclution

Mathematic predator-prey modeling always involves interactions that occur in ecosystems. Models that are prepared with strong basic assumptions will become a simulation model that is capable of becoming policy formulations or solving ecosystem problems. Model (1, 2, 3) is the basis of the predator-prey model involving a Ratio-Dependent type of interaction.The proposed mathematical model is also given an exploitation effort. The exploitation provided is to involve harvesting efforts in each population differently.The model is constructed by analyzing local asymptotic equilibrium points, ie non-negative $E_1(x_1,z_1)$. The equilibrium point is obtained based on the linearization of the differential equation solutions. The next test on the equilibrium point is the fulfillment of the Routh-Hurwitz criteria. Tests were carried out to demonstrate point local asystotic stability $E_1(x_1,z_1)$. The eigenvalue analysis corresponding to the model supports a stable form of the mathematical model.

Trajectories analysis is also given to the proposed model. The characteristics that emerge from each species are different. Trajectories analysis is used to confirm the mathematical data that has been obtained. Movement of predator and prey population growth without involving harvesting fulfills a stable condition for a long period of time. Therefore, exploitation can be done. The first exploitation was carried out on prey species only, with profitable exploitation results for business actors. From the profit function, the total income earned is $\psi(H_1)=7677.782008$. Exploitation efforts on prey species are considered successful, because they provide local asymptotic stable points that meet the Routh-Hurwitz criteria. Meanwhile, in the business of exploiting predatory species, it provides a total profit of $\psi(H_2)=123.7573838$. The same condition is also shown at the point of equilibrium and continuous exploitation. Economically both businesses provide maximum profit. When compared, the business of harvesting exploitation of prey and predators, harvesters must choose a form of harvesting prey, with greater profits and a stable growth rate of species over a very long period of time.The results of this study can be used as reference material or consideration for further research. Exploitation business development with a more complex harvest can be added. The response function model can also be an option for developing the basic Ratio-Dependent model.

References

[1] R. A. Pratama, M. Loupatty, H. Hariyanto, W. Caesarendra, and W. Rahmaniar, “Fear and Group Defense Effect of a Holling Type IV Predator-Prey System Intraspecific Competition,” *Emerg. Sci. J.*, vol. 7, no. 2, pp. 385–395, 2023, doi: 10.28991/ESJ-2023-07-02-06.

[2] M. Liu and C. Bai, “Optimal harvesting policy for a stochastic predator-prey model,” *Appl. Math. Lett.*, vol. 34, no. 1, pp. 22–26, 2014, doi: 10.1016/j.aml.2014.03.006.

[3] V. Madhusudanan, M. N. Srinivas, and B. R. Tapas Bapu, “Influence of predator standby capacity, harvesting and noise on a two patchy aquatic delayed eco system with migration of prey,” *Informatics Med. Unlocked*, vol. 13, no. April, pp. 158–166, 2018, doi: 10.1016/j.imu.2018.06.001.

[4] A. Mondal, A. K. Pal, and G. P. Samanta, “Complex dynamics of two prey–one predator model together with fear effect and harvesting efforts in preys,” *J. Comput. Math. Data Sci.*, vol. 6, no. August 2021, p. 100071, 2023, doi:

10.1016/j.jcmds.2022.100071.
[5] M. Ibrahim, “Optimal harvesting of a predator-prey system with marine reserve,” *Sci. African*, vol. 14, p. e01048, 2021, doi: 10.1016/j.sciaf.2021.e01048.
[6] S. Hossain, M. M. Haque, M. H. Kabir, M. O. Gani, and S. Sarwardi, “Complex spatiotemporal dynamics of a harvested prey–predator model with Crowley–Martin response function,” *Results Control Optim.*, vol. 5, no. October, p. 100059, 2021, doi: 10.1016/j.rico.2021.100059.
[7] A. K. Pal, A. Bhattacharyya, and A. Mondal, “Qualitative analysis and control of predator switching on an eco-epidemiological model with prey refuge and harvesting,” *Results Control Optim.*, vol. 7, no. February, p. 100099, 2022, doi: 10.1016/j.rico.2022.100099.
[8] X. Gao, S. Ishag, S. Fu, W. Li, and W. Wang, “Bifurcation and Turing pattern formation in a diffusive ratio-dependent predator–prey model with predator harvesting,” *Nonlinear Anal. Real World Appl.*, vol. 51, p. 102962, 2020, doi: 10.1016/j.nonrwa.2019.102962.
[9] A. A. Thirthar, S. J. Majeed, K. Shah, and T. Abdeljawad, “The dynamics of an aquatic ecological model with aggregation, Fear and Harvesting Effects,” *AIMS Math.*, vol. 7, no. 10, pp. 18532–18552, 2022, doi: 10.3934/math.20221018.
[10] R. A. Pratama, S. Toaha, and Kasbawati, “Optimal harvesting and stability of predator prey model with Monod-Haldane predation response function and stage structure for predator,” *IOP Conf. Ser. Earth Environ. Sci.*, vol. 279, no. 1, 2019, doi: 10.1088/1755-1315/279/1/012015.
[11] Ö. A. Gümüş, “Dynamics of a Prey-Predator System with Harvesting Effect on Prey,” *Chaos Theory Appl.*, vol. 4, no. 3, pp. 144–151, 2022, doi: 10.51537/chaos.1183113.
[12] H. Reşit Akçakaya, R. Arditi, and L. R. Ginzburg, “Ecological Society of America Ratio-Dependent Predation: An Abstraction That Works,” *Source Ecol.*, vol. 76, no. 3, pp. 995–1004, 1995.
[13] B. Mukhopadhyay and R. Bhattacharyya, “Effects of harvesting and predator interference in a model of two-predators competing for a single prey,” *Appl. Math. Model.*, vol. 40, no. 4, pp. 3264–3274, 2016, doi: 10.1016/j.apm.2015.10.018.
[14] Y. V. Tyutyunov and L. I. Titova, “Ratio-Dependence in Predator-Prey Systems as an Edge and Basic Minimal Model of Predator Interference,” *Front. Ecol. Evol.*, vol. 9, no. December, 2021, doi: 10.3389/fevo.2021.725041.
[15] R. P. Gupta and P. Chandra, “Bifurcation analysis of modified Leslie-Gower predator-prey model with Michaelis-Menten type prey harvesting,” *J. Math. Anal. Appl.*, vol. 398, no. 1, pp. 278–295, 2013, doi: 10.1016/j.jmaa.2012.08.057.
[16] M. Li, B. Chen, and H. Ye, “A bioeconomic differential algebraic predator–prey model with nonlinear prey harvesting,” *Appl. Math. Model.*, vol. 42, pp. 17–28, 2017, doi: 10.1016/j.apm.2016.09.029.
[17] X. Wang and Y. Wang, “Novel dynamics of a predator–prey system with harvesting of the predator guided by its population,” *Appl. Math. Model.*, vol. 42, pp. 636–654, 2017, doi: 10.1016/j.apm.2016.10.006.